



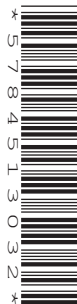
Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



1 (a) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^n (2-3r)(5-3r) = an^3 + bn^2 + cn,$$

where a, b and c are integers to be determined.

[3]

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN





- (b) Use the method of differences to find $\sum_{r=1}^n \frac{1}{(2-3r)(5-3r)}$ in terms of n . [4]

[illegible]

- (c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(2-3r)(5-3r)}$. [1]

.....





2 The cubic equation $x^3 + 2x + 1 = 0$ has roots α, β, γ .

(a) Find a cubic equation whose roots are $\alpha^3 - 1, \beta^3 - 1, \gamma^3 - 1$. [3]

[3]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(b) Find the value of $(\alpha^3 - 1)^2 + (\beta^3 - 1)^2 + (\gamma^3 - 1)^2$.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the value of $(\alpha^3 - 1)^3 + (\beta^3 - 1)^3 + (\gamma^3 - 1)^3$.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

* 0000800000007 *



7

BLANK PAGE

DO NOT WRITE IN THIS MARGIN





- (a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x - y plane.

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings present.

-
- This image shows a full page of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the entire width of the page. There is no handwriting or other markings on the paper.

[illegible]



10

- 5 The curve C has polar equation $r = \theta e^{\frac{1}{8}\theta}$, for $0 \leq \theta \leq 2\pi$.

(a) Sketch C .

[2]

- (b) Find the area of the region bounded by C and the initial line, giving your answer in the form $(p\pi^2 + q\pi + r)e^{\frac{1}{2}\pi} + s$, where p, q, r and s are integers to be determined. [6]

[illegible]



- (c) Show that, at the point of C furthest from the initial line,

$$\theta \cos \theta + \left(\frac{1}{8}\theta + 1\right) \sin \theta = 0$$

and verify that this equation has a root between 5 and 5.05.

[5]





6 The points A, B, C have position vectors

$$\mathbf{i} - 2\mathbf{k}, \quad \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} - \mathbf{j} - \mathbf{k},$$

respectively.

(a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

A point D has position vector $\mathbf{i} + t\mathbf{k}$, where $t \neq -2$.

(b) Find the acute angle between the planes ABC and ABD . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





(c) Find the values of t such that the shortest distance between the lines AB and CD is $\sqrt{2}$.

[7]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.



[2]

[illegible]

[4]

[illegible]

(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

(d) Sketch the curve with equation $y = \left| \frac{2x^2 - 5x}{2x^2 - 7x - 4} \right|$.

[1]





[5]

[illegible]



Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

* 0000800000018 *



BLANK PAGE



DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN



* 0000800000019 *



BLANK PAGE

PMT



* 0000800000020 *



20

BLANK PAGE

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

